



## PROBLEM SET 2

1 In rectangle  $ABCD$ , let  $M$  and  $N$  be the midpoints of  $BC$  and  $CD$ , respectively. Let  $DM$  and  $BN$  intersect at  $P$ .

Prove that  $\angle MAN = \angle BPM$ .

2 A triangle  $ABC$  has squares  $PABK$  and  $QACL$  constructed on its exterior. The altitude  $AD$  of triangle  $ABC$  is extended to meet  $PQ$  at point  $M$ .

Prove that  $M$  is the midpoint of  $PQ$ .

3 Suppose that  $N$  is the midpoint of the side  $BC$  of triangle  $ABC$ . Construct right isosceles triangles  $AMB$  and  $APC$  on sides  $AB$  and  $AC$  outside the triangle where,  $\angle AMB = \angle APC = 90^\circ$ .

Prove that  $MNP$  is also a right isosceles triangle.

4 Let  $ABD$  be a triangle and let  $C$  be a point on the side  $BD$ , lying strictly between  $B$  and  $D$ . Suppose that  $BC = 2CD$ ,  $\angle ACB = 60^\circ$  and  $\angle ADC = 45^\circ$ .

Determine  $\angle BAD$ .

5 Let  $P$  be a point on the circumcircle of triangle  $ABC$ . Let  $D$ ,  $E$  and  $F$  be the feet of the perpendiculars from  $P$  to the lines  $BC$ ,  $AC$  and  $AB$ , respectively.

Show that  $D$ ,  $E$  and  $F$  are collinear.

6 Let  $P$  be a point in the plane of a triangle  $ABC$ . Reflect the lines  $PA$ ,  $PB$  and  $PC$  through the angle bisectors at  $A$ ,  $B$  and  $C$ , respectively.

Prove that these three reflected lines are concurrent.