PROBLEM SET 1

1 Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that $f(x) \neq 0$ for $x \neq 0$ and

$$f(f(x) + y) = f(x^2 - y) + 4f(x)y$$

for all real numbers x and y.

2 Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that

$$f(x - f(y)) = 1 - x - y$$

for all real numbers x and y.

3 Find all bijections $f : \mathbb{R} \to \mathbb{R}$ such that f is increasing and

$$f(x) + f^{-1}(x) = 2x$$

for every real number x.

4 Find all functions $f : \mathbb{Z}^+ \to \mathbb{Z}^+$ such that

$$f(n + 1) > f(f(n))$$

for all real numbers x and y.

5 Let $\mathcal{G}$ be a class of functions $\mathbb{R} \to \mathbb{R}$ of the form $f(x) = ax + b$, where $a \neq 0$. The class $\mathcal{G}$ is closed under taking inverses and under composition of functions. Now suppose that for each $f \in \mathcal{G}$ there is a point fixed by f. Prove that the functions in $\mathcal{G}$ have a common fixed point.

6 Show that there is no function $f : \mathbb{Z} \to \mathbb{Z}$ satisfying

$$f(f(n)) = n + 1$$

for every integer n.