



Competitive
Programming and
Mathematics
Society

UNSW CPMSOC WORKSHOP 1

FEBRUARY 26, 2021

Proof By Contradiction



PROBLEMS

- A1 Show that $\log_{10}(2)$ is irrational.
- A2 Show that there does not exist a function $f : \mathbb{Z} \rightarrow \{1, 2, 3\}$ satisfying $f(x) \neq f(y)$ for all $x, y \in \mathbb{Z}$ such that $|x - y| \in \{2, 3, 5\}$.
- A3 Every point of three-dimensional space is coloured red, green or blue. Show that one of the colours attains all distances; that is, any positive real number represents the distance between two points of this colour.
- A4 Show that no set of nine consecutive integers can be partitioned into two sets with the product of the elements of the first set equal to the product of the elements of the second set.
- A5 Prove that infinitely many primes are one more than a multiple of 4.
- A6 Given any sequence of $mn + 1$ real numbers, show that some subsequence of length $(m + 1)$ is increasing or some subsequence of length $(n + 1)$ is decreasing.
- A7 Show that there does not exist a strictly increasing function $f : \mathbb{N} \rightarrow \mathbb{N}$ satisfying $f(2) = 3$ and $f(mn) = f(m)f(n)$ for all $m, n \in \mathbb{N}$.